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Multi-dimensional Opinion Chaos Synchronization Based on Multi-layer Complex Networks

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Abstract—**In this paper we discuss the dynamics of opinion evolution on multi-layer complex networks. A new multi-dimensional opinion evolution model combining multi-layer complex network and Lorenz chaotic system is investigated for opinion synchronization of multi-layer complex networks. The model can be attached to multiple event opinions and simulate the evolution dynamics of the multi-dimensional opinions under natural conditions. Based on the Lyapunov function, a new criterion for opinion synchronization of multi-layer complex networks is proposed. And the reliability of the model and criteria is verified by the Runge-Kutta (RK4) simulation.**

***Keywords-** Complex Networks; Synchronization; Lyapunov function; Runge-Kutta (RK4);*

I. Introduction

Synchronization as an important dynamic behavior in complex networks and systems, which has received extensive attention due to its wide application in image encryption, signal processing, and public transportation. Recently, some achievements have been made in the synchronization research of complex networks.

Yu and Chen et al. studied the synchronization problem of pinning control in general complex dynamic networks, and gave a criterion to ensure network synchronization in strongly connected networks [1]. Furthermore, Guo and Li solved the synchronization problem of stochastic complex networks with time-varying delays by aperiodic intermittent control (AIC) [2]. In addition, Liu and Chen studied the exponential synchronization of linear coupled networks by fixing a simple aperiodic intermittent controller [3], and designed an adaptive algorithm for pinning control [4]. Tian and Ren proposed an impulsive control method, which is for the synchronization of two linear or nonlinear hyper-chaotic Chen circuits [5][6]. Das Abhijit and Frank L. Lewis studied the synchronization of distributed node dynamics to a prescribed target or control node dynamics, and an adaptive synchronous controller is proposed for the distributed systems with non-identical unknown nonlinear dynamics [7]. Zheng S investigated the adaptive-impulsive projective synchronization of drive-response delayed complex dynamical networks, and designed an adaptive feedback controller with impulsive control effects [8].

People play different roles in social networks and express or receive different opinions. The process of public opinion in social networks has been a hot research topic in the field of complex networks. Scholars have also done a lot of research on the synchronization of opinions in complex social networks, and various dynamic models of opinions have been developed in recent years. For example, reference [9] put forward the Ising model according to the characteristics of individuals (nodes in the network), while the reference [10] use the physical phenomena to analogize the process of opinion formation and put forward the Sznajd model. Due to the rapid popularization of the Internet, the dissemination of information and public opinion are more complicated and rapid than decades before. The SIR model and the SIS model, which are originally used to simulate the process of epidemic-infected population dynamics, have also been introduced to simulate the dynamics of opinion dissemination [11] [12]. The Hegselmann and Krause (HK) model proposed in reference [13] is based on the concept of bounded confidence. In the HK model, if the difference between the opinions of two agent nodes is less than ǫ, it means that their positions are close, and their opinions will change with the discussion [14].

In fact, the research on the synchronization of opinions is need to combine with the complex social network model and the dynamic model of opinion evolution, but most of the current research on the complex social network model mainly focus on single-layer networks, ignoring the complexity of social networks and the multiplicity of social roles. Moreover, many current opinion evolution models have the limitation of neither taking into account the easily affected and changed feature of opinions, nor the multi-opinion feature of individual nodes.

Based on the above depiction, in this paper, a new multi-dimensional opinion evolution model is investigated for opinion synchronization of multi-layer complex networks. The main contributions of this paper are listed: (i) The evolution and synchronization of multi-dimensional opinion in complex networks is studied. (ii) Multi-layer complex network model and Lorenz chaos model is combined to simulate the evolution dynamics of the multi-dimensional opinions under natural conditions. (iii) A new criteria for opinion synchronization of multi-layer complex networks is proposed.

The rest of the paper is organized as follows. Firstly, Sec. II presents a mathematical model that can not only show the model structure of multi-layer complex social networks, but also the evolution dynamics of multi-dimensional opinion. In Sec. III,

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the assumptions, definitions and related lemmas for opinion synchronization are shown. Next, the numerical example and simulations are illustrated in Sec. IV. Finally, some concluding remarks are given in Sec. V.

II. Model

A. Multi-layer Social Network

Social network was formally defined as a communication network composed of a set of network members and intricate relationships between members [15]. From the perspective of complex networks, each network member can be abstracted as a node, and the network member's previous social relationship can be regarded as edge in the network graph. For example, the friend relationship in WeChat, and mutual fans relationship in Weibo.

In real life, a person may play different roles and have one or more types of social relationships on social platforms, so the process of dissemination and synchronization of public opinions will become more complex. The simplest example for a topic contains an offline communication network and an online communication network. As the sketch map shown in Figure 1. , different label numbers of nodes represent different people, and a pair of nodes with the same label number represent different roles played by the same person in offline and online networks. The edges between nodes in the two-layer network represent social relationships. It can be seen from the figure that the social relationships of nodes in the offline network and the online network are different.

In order to simplify the complex network model, we give it the definition of graph. Taking Figure 1. as an example, we assume that $G_{\mathrm{on}} = (V_1, E_1)$ and $G_{\mathrm{off}} = (V_1, E_2)$ are simple undirected graphs, where $V_1 = \{v_1, \dots, v_n\}$. Then the entire complex multi-layer network $G_{\mathrm{multi}}(V, E)$ can be defined by $V = V_1 = V_2$ and $E = E_1 \cup E_2$. We can consider $\mathbf{x_i} = [x_i, y_i, z_i]^T$ as three-dimensional vectors assigned to a node i in V_1.

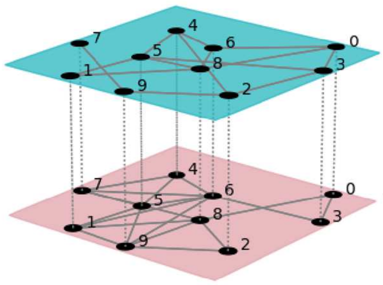

Figure 1. A sketch map for the Multilayer Social Network

B. Chaos Opinion Evolution Model

Many current opinion evolution models have the limitation of neither taking into account the easily affected and changed feature of opinions, nor the multi-opinion feature of individual nodes. In fact, due to complex social network relationships, each of us may change our opinions, perspectives and ways of thinking according to the information from the Internet or the life. The difference is that some conservative people may stick to their own opinions for a long time, while there are also some flexible minds who follow the current fashions and trends and easily change their opinions. These different tendencies is similar to the fluctuation spectrum of oscillators in a chaotic system, and its evolution results have an unknowable characteristic as well.

This tendency was combined with the Kuramoto chaotic model in 2006 and a new concept called *Natural Opinion Changing Rates (OCR)* was proposed to represent the intrinsic inclinations of the individuals to change their opinion [16]. In this paper, considering the multi-dimensional characteristics of individual opinions, we adopt the Lorenz three-dimensional oscillation model to represent the multi-dimensional opinion evolution state of network nodes in the natural conditions. The equation of the Lorenz oscillation model is described by:

$$\begin{cases} \dot{x} = \sigma(y - x) \\ \dot{y} = \rho x - y - xz \\ \dot{z} = xy - \beta z \end{cases} \tag{1}$$

where σ, ρ, β are model parameters.

When σ=10, ρ=28, β=8/3, the three-dimensional evolution trajectory of the Lorenz model, which start from the initial values x=-16, y=-21, z=33 is shown in the Figure 2. It can be seen that the system only moves in a limited area of the three-dimensional space.

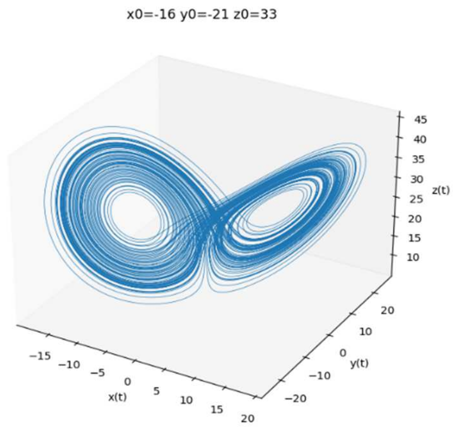

Figure 2. The three-dimensional evolution trajectory of the Lorenz model (initial values x=-16, y=-21, z=33)

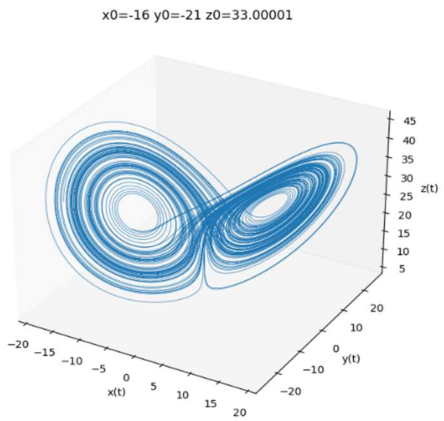

Figure 3. The three-dimensional evolution trajectory of the Lorenz model (initial values x=-16, y=-21, z=33.00001)

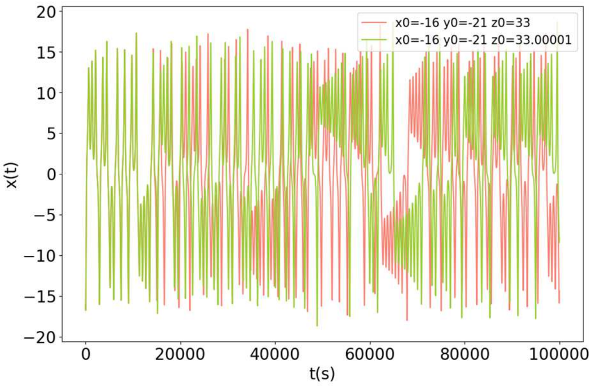

Figure 4. The deviation evolution curve of $x(t)$ with the Lorenz system starts from two very close initial values

Furthermore, if the initial value of the Lorenz model changes slightly (with a difference of only 0.00001), compared with Figure 3. , the evolution trajectories will be quite different. This can be seen more clearly from Figure 4. , which shows the deviation evolution curve of $x(t)$. Trajectories that were close together then quickly separated over time, and two trajectories became unrelated at the end. The whole result intuitively reflects the sensitivity of the chaos model to the initial value.

C. *Multi-layer complex network opinion evolution model*

Combining the multilayer social network model with the chaos opinion evolution model, we consider a multi-layer opinion evolution model of complex network with N identical nodes, in which each node is an n-dimensional dynamical system. The state equations of the model as follows:

$$\dot{\mathbf{x}}_i(\mathrm{t}) = \mathbf{f}_i\big(\mathbf{x}_i(\mathrm{t}), \mathbf{x}_i(\mathrm{t}-\tau)\big) - \sum_{\alpha=1}^{M} \sigma_\alpha \sum_{j=1}^{N} L^\alpha H \mathbf{x}_j(\mathrm{t}) \quad (2)$$

where $\mathbf{x}_i(\mathrm{t}) = [x_i(t), y_i(t), z_i(t)]^T$ represents state variables of node i that changes with time t. $i = 1,2, \dots, N$. M is the number of network layer. σ_α represents the coupling strength of the α-th layer network and H is the coupling matrix of the network nodes. L^α represents the Laplacian matrix of the α-th layer network. In the Laplace matrix, if two nodes are not connected, then l_{ij}=0, otherwise, l_{ij}=-1.

Because the first part $\mathbf{f}_i\big(\mathbf{x}_i(\mathrm{t}), \mathbf{x}_i(\mathrm{t}-\tau)\big) = \Big(f\big(x_i(t), x_i(t-\tau)\big), f\big(y_i(t), y_i(t-\tau)\big), f\big(z_i(t), z_i(t-\tau)\big)\Big)^T$ of equation (2) is a continuous nonlinear vector-valued function, which represents the multi-dimensional opinion evolution dynamics of network nodes under natural conditions. Likewise, the latter part $\sum_{\alpha=1}^{M} \sigma_\alpha \sum_{j=1}^{N} L^\alpha H \mathbf{x}_j(\mathrm{t})$ of equation (2) involves the topology structure and coupling strength of the multilayer network, which represents the evolution dynamics of the nodes' opinions under the influence of the network structure. Therefore, the evolution dynamics of opinions under the influence of multi-layer complex network can be simulated by the whole model equation (2)

Replace the chaos model in equation (2) with the Lorenz oscillation model, and take the parameters σ=10, ρ=28, β=8/3, then the final opinion evolution model equation of multi-layer complex network is:

$$\begin{cases} \dot{x}_i(t) = 10(y_i - x_i) - \sum_{\alpha=1}^{M} \sigma_\alpha \sum_{j=1}^{N} L^\alpha H x_j \\ \dot{y}_i(t) = 28x_i - y_i - x_i z_i - \sum_{\alpha=1}^{M} \sigma_\alpha \sum_{j=1}^{N} L^\alpha H y_j \\ \dot{z}_i(t) = x_i y_i - 8/3 z_i - \sum_{\alpha=1}^{M} \sigma_\alpha \sum_{j=1}^{N} L^\alpha H z_j \end{cases} \quad (3)$$

During the synchronous simulation of the network, the opinion attributes related to three different social events are attached to the nodes' three sub-vectors respectively, so as to represent the evolution of individual nodes' opinions and attitudes towards multiple events.

III. Main Results

Take the three social events that have been widely discussed on Weibo, including the "HuoLala female passenger's death event", "7.23 Beijing Badaling Wildlife Park tiger wounding event", and "Jiangge-Liu Xin case", for example. Each event represents an opinion dimension, and then opinion synchronization can be defined as follows:

Assumption 1: Under the initial condition $\varphi(t) \in C([-\tau, 0], R^n)$, Assume that $s(t, \tau; \varphi(t))$ is a solution of an individual node about the network system (2) satisfying

$$\dot{s}(t) = f\big(s(t), s(t-\tau)\big) \quad (4)$$

where $s\big(t, \tau; \varphi(t)\big)$ may be a nontrivial periodic orbit and represent the evolution process of the node's opinions on these three events during time t.

Definition 1: Under the initial condition $\psi(t) \in C([-\tau, 0], R^n)$, let $\mathbf{x}_i\big(t, \tau; \psi(t)\big)(1 \leqslant i \leqslant N) = [x_i\big(t, \tau; \psi(t)\big), y_i\big(t, \tau; \psi(t)\big), z_i\big(t, \tau; \psi(t)\big)]^T$ be the a set of solutions for network system(3), representing the evolution of the total N nodes' opinions on these three events. If there are arbitrary initial conditions $\psi(t), \varphi(t) \in C([-\tau, 0], R^n)$ satisfying any of the following formulas:

$$\lim_{t\to\infty} \|x_i\big(t, \tau; \psi(t)\big) - s_x\big(t, \tau; \varphi(t)\big)\| = 0,\ 1 \leqslant i \leqslant N \quad (5)$$

$$\lim_{t\to\infty} \|y_i\big(t, \tau; \psi(t)\big) - s_y\big(t, \tau; \varphi(t)\big)\| = 0,\ 1 \leqslant i \leqslant N \quad (6)$$

$$\lim_{t\to\infty} \|z_i\big(t, \tau; \psi(t)\big) - s_z\big(t, \tau; \varphi(t)\big)\| = 0,\ 1 \leqslant i \leqslant N \quad (7)$$

then the controlled opinion evolution is said to be asymptotically synchronized in a certain dimension, that is, single-dimensional synchronization, which means that the opinions of these individual nodes on a certain event are consistent. If the three equations are satisfied at the same time, then the network is said to be fully synchronized, that is, the total N nodes have the same opinions on the three events.

From the above definition of opinion synchronization, we can give the error definitions of single-dimensional synchronization and full synchronization of opinions:

Definition 2: Define single-dimensional error vectors as:

$$e_x(t) = \left(\frac{1}{N(N-1)} \sum_{i,j} \|x_i(t) - x_j(t)\|^2 \right)^{\frac{1}{2}} \quad (8)$$

$$e_y(t) = \left(\frac{1}{N(N-1)} \sum_{i,j} \|y_i(t) - y(t)\|^2 \right)^{\frac{1}{2}} \quad (9)$$

$$e_z(t) = \left(\frac{1}{N(N-1)} \sum_{i,j} \|z_i(t) - z_j(t)\|^2 \right)^{\frac{1}{2}} \quad (10)$$

If all individual nodes agree on an event, then one of the following corresponding formulas is satisfied:

$$\lim_{t\to\infty} e_x(t) = 0 \quad (11)$$

$$\lim_{t\to\infty} e_y(t) = 0 \quad (12)$$

$$\lim_{t\to\infty} e_z(t) = 0 \quad (13)$$

Definition 3: Consider the following error to judge the evolution state of all opinions:

$$e_i(t) = \left(\frac{1}{N(N-1)} \sum_{i,j} \|\mathbf{x}_i(t) - \mathbf{x}_j(t)\|^2 \right)^{\frac{1}{2}} \quad (14)$$

$\|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2 + (z_i - z_j)^2}$ is the Euclidean norm. When all individual nodes agree on the three events, then $\lim_{t\to\infty} e_i(t) = 0$.

IV. SIMULATION

In this section, we use the "Bernard & Killworth fraternity network" with a two-layer structure as an example to verify the correctness of the above theoretical results. The diagram of the network model structure is as follows:

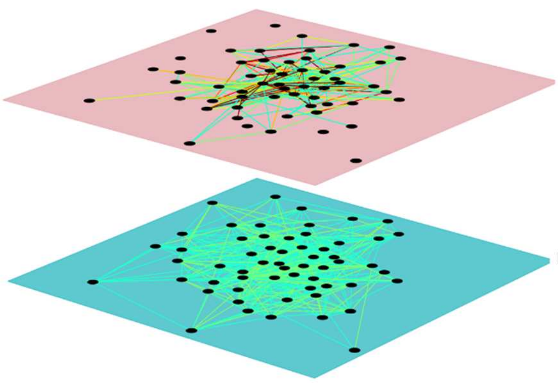

Figure 5. The "Bernard & Killworth fraternity network" model structure diagram

The network is constructed from human interaction data collected by Bernard and Killworth, and the edges in the network represent interactions among the fifty-eight students living in a fraternity at a West Virginia University. All subjects were members of the fraternity, and had been residents in the fraternity from three months to three years. BKFRAB recorded the number of times two topics were discussed at a talking activity of the fraternity, and BKFRAC records the number of conversations these people have with other people in the fraternity during the week. We take the number of four times as the threshold.

It can be known from the calculation that the coupling coefficient of the BKFRAB layer network is σ_1=0.4834, and the maximum eigenvalue of its Laplacian matrix is λ_1^{max}=53.064 the coupling coefficient of the BKFRAB layer network is σ_2=0.5927, and the largest eigenvalue of its Laplacian matrix is λ_2^{max}=58.

In simulation, we take N=58, M=2, the maximum coupling coefficient and the maximum eigenvalue are $\sigma_{\max}$=0.5927 and λ^{max}=58. The Runge-Kutta (RK4) method was adopted for our simulations and took the time step t as 0.02 and the delay time τ as 0.08. We can obtain the oscillation curve and error curve of the network node as follows:

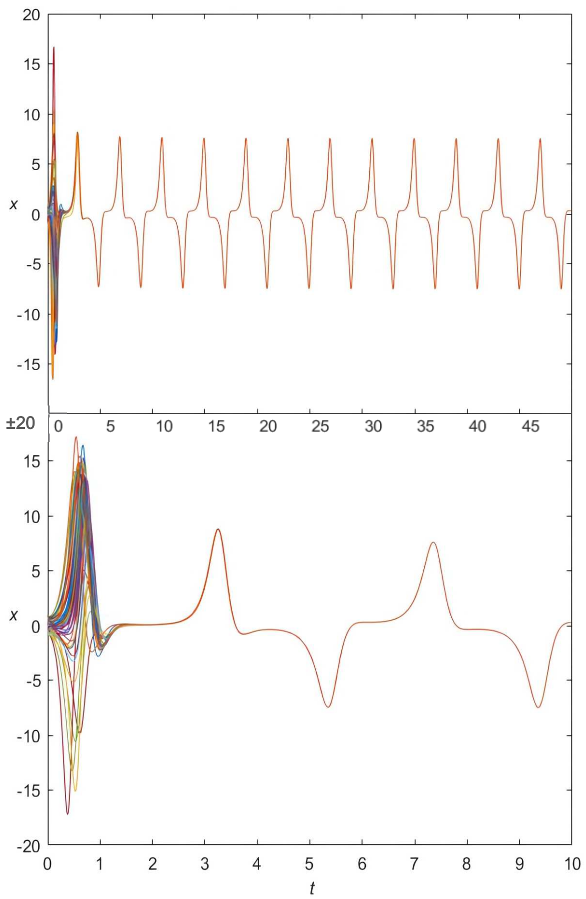

Figure 6. Oscillation curve of $x_i(t)$

Figure 6. shows the evolution trajectories of fifty-eight student nodes from different initial values. It can be clearly seen that the evolution trajectories will are quickly synchronized and oscillate at the same frequency.

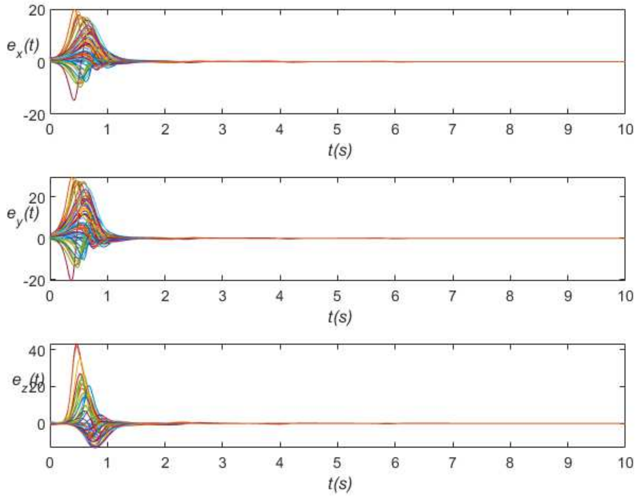

Figure 7. Synchronization errors curve

Figure 7. shows the opinion synchronization errors of fifty-eight student nodes. Clearly, errors in all dimensions are rapidly converging to zero.

V. CONCLUSION

In this paper, a new multi-dimensional opinion evolution model combining multi-layer complex network and Lorenz chaotic system has been investigated. Firstly, since the oscillation frequency of Lorenz chaotic model is similar to the natural Opinion Rate of Change (OCR), this model can more realistically simulate the evolution dynamics of the multi-dimensional opinions under natural conditions. Then, the model can be attached to multiple event opinions and simulate the evolution dynamics of the multi-dimensional opinions under natural conditions. Based on the Lyapunov function, a new criterion for opinion synchronization of multi-layer complex networks is proposed. Finally, numerical simulations using Runge-Kutta (RK4) have been presented to illustrate the effectiveness of the proposed synchronization criteria.

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